Enhancing Students' Reasoning Through Teacher Questioning (F - Y6)

Faculty of Education and Arts
NSW School of Education

Dr Chrissy Monteleone

Deputy Head of School

Chrissy.monteleone@acu.edu.au

December 2024

Overview

1.0

- Positioning explicit teaching within this presentation

2.0

- Mathematical reasoning

3.0

- Teacher questioning

Is this presentation about explicit teaching?

How a more explicit teaching style helped this school defy NAPLAN struggles elsewhere

By national education and parenting reporter Conor Duffy and the Specialist Reporting Team's Evan Young
Posted Thu 24 Aug 2023 at 4:59am

Nasya Hassan is a teacher at Sydney Adventist School, where NAPLAN results have seen a big turnaround. *(ABC News: Marcus Stimson)*

This was published 6 months ago

Education boss calls for doubling down on explicit teaching in schools

Lucy Carroll
October 23, 2023 – 5.00am

Save Share A A A 76 View all comments

Listen to this article
6 min

The head of the NSW public education system has called on schools to double down on the use of explicit instruction – a teaching method that gives students step-by-step and clear instructions – in a bid to boost results and close the stark achievement gap.

ACU
AUSTRALIAN CATHOLIC UNIVERSITY

How an explicit teaching approach optimises learning

Breaking down and fully explaining content helps students transfer information to memory

Providing opportunities for review and practise helps students retain and recall what they learn

Organising and sequencing content around a specific objective deepens student understanding

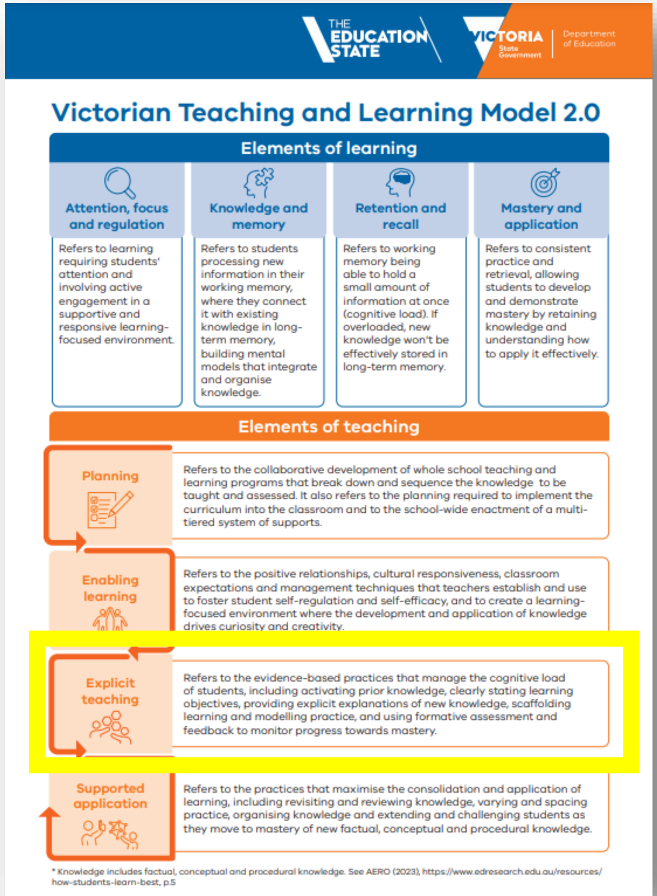
Knowledge-building, guidance and scaffolding supports students with additional limitations in prior knowledge, memory and information processing

Structure and guidance help novices develop mastery over their learning

A strong foundation of knowledge and skills provides students with mental models for extending and applying their learning with greater independence

Australian Education Research Organisation, 2023

ACU
AUSTRALIAN CATHOLIC UNIVERSITY



THE EDUCATION STATE | VICTORIA State Government | Department of Education

Victorian Teaching and Learning Model 2.0

Elements of learning

Attention, focus and regulation	Knowledge and memory	Retention and recall	Mastery and application
Refers to learning requiring students' attention and involving active engagement in a supportive and responsive learning-focused environment.	Refers to students processing new information in their working memory, where they connect it with existing knowledge in long-term memory, building mental models that integrate and organise knowledge.	Refers to working memory being able to hold a small amount of information at once (cognitive load). If overloaded, new knowledge won't be effectively stored in long-term memory.	Refers to consistent practice and retrieval, allowing students to develop and demonstrate mastery by retaining knowledge and understanding how to apply it effectively.

Elements of teaching

Planning	Refers to the collaborative development of whole school teaching and learning programs that break down and sequence the knowledge to be taught and assessed. It also refers to the planning required to implement the curriculum into the classroom and to the school-wide enactment of a multi-tiered system of supports.
Enabling learning	Refers to the positive relationships, cultural responsiveness, classroom expectations and management techniques that teachers establish and use to foster student self-regulation and self-efficacy, and to create a learning-focused environment where the development and application of knowledge drives curiosity and creativity.
Explicit teaching	Refers to the evidence-based practices that manage the cognitive load of students, including activating prior knowledge, clearly stating learning objectives, providing explicit explanations of new knowledge, scaffolding learning and modelling practice, and using formative assessment and feedback to monitor progress towards mastery.
Supported application	Refers to the practices that maximise the consolidation and application of learning, including revisiting and reviewing knowledge, varying and spacing practice, organising knowledge and extending and challenging students as they move to mastery of new factual, conceptual and procedural knowledge.

* Knowledge includes factual, conceptual and procedural knowledge. See AERO (2023), https://www.edresearch.edu.au/resources/how-students-learn-best, p.5

Victorian State Government, 2023

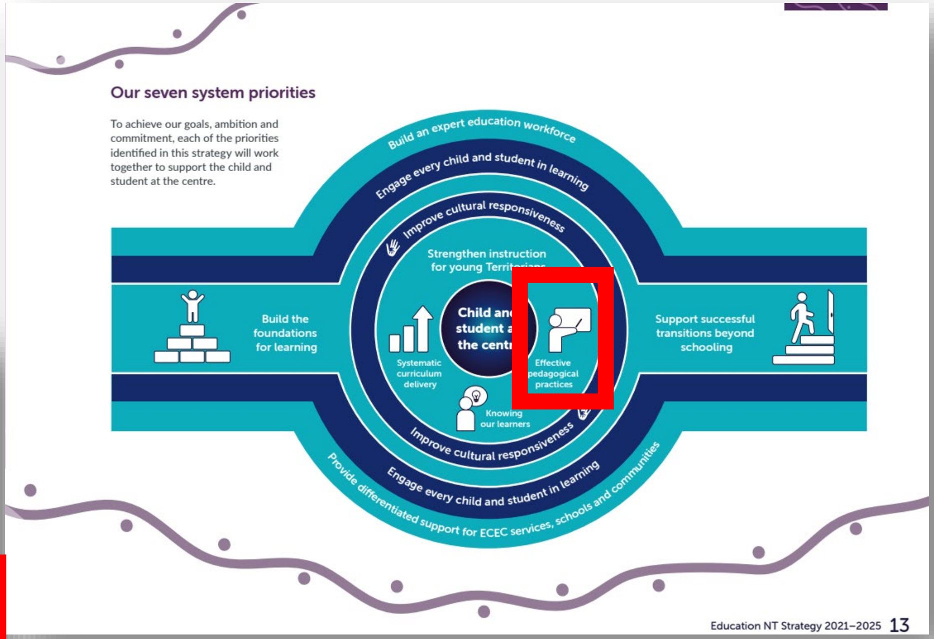
NSW Department of Education

NSW GOVERNMENT

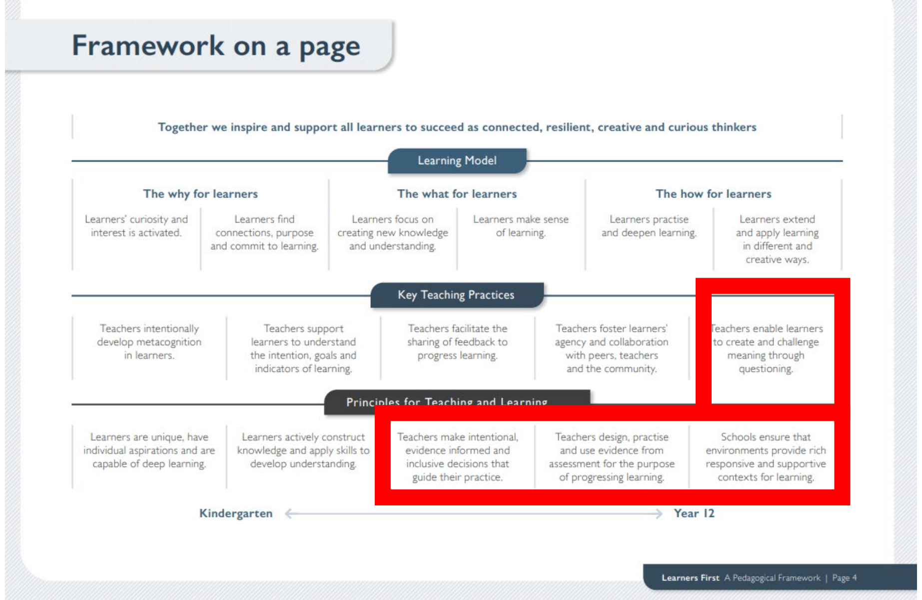
Dimensions

Intellectual Quality	Quality Learning Environment	Significance
Intellectual Quality refers to pedagogy focused on deep understanding of important, substantive concepts, skills and ideas. Such pedagogy treats knowledge as requiring active construction and engages students in higher-order thinking and communicating about what they are learning.	Quality Learning Environment refers to pedagogy that creates classrooms where students and teachers work productively and are clearly focused on learning. Such pedagogy sets high expectations and develops positive relationships among teachers and students.	Significance refers to pedagogy that helps make learning meaningful to students. Such pedagogy draws clear connections with students' prior knowledge and identities, with contexts outside of the classroom, and multiple ways of knowing or cultural perspectives.
Deep Knowledge Knowledge is deep when it concerns the central ideas or concepts of a topic, subject or learning area and when the knowledge is judged to be crucial to the topic, subject or learning area.	**Explicit Quality Criteria** High explicit quality criteria is identified by frequent, detailed and specific statements about the quality of work required of students.	**Background Knowledge** High background knowledge is evident when lessons provide students with opportunities (or they take opportunities) to make connections between their knowledge and experience and the substance of the lesson.
Deep Understanding Deep understanding is evident when students demonstrate their grasp of central ideas and concepts.	**Engagement** High engagement is identified by on-task behaviours that signal a serious investment in class work.	**Cultural Knowledge** Cultural knowledge is high when there is an understanding, valuing and acceptance of the traditions, beliefs, skills, knowledges, languages, practices and protocols of diverse social groups.
Problematic Knowledge Knowledge is treated as problematic when it involves an understanding of knowledge not as a fixed body of information, but rather as socially constructed, and hence subject to political, social and cultural influences and implications.	**High expectations** Expectations are high when teachers (or students) communicate the expectation that all members of the class can learn important knowledge and skills that are challenging for them.	**Knowledge Integration** High knowledge integration is identifiable when meaningful connections are made between different topics and/or between different subjects.
Higher-Order Thinking Higher-order thinking requires students to manipulate information and ideas in ways that transform their meaning and implications.	**Social Support** Classrooms high in social support for student learning encourage all students to try hard and risk initial failure in a climate of mutual respect.	**Inclusivity** High inclusivity is evident when all students, from all cultural or social backgrounds, participate in the public work of the class and when their contributions are taken seriously and valued.
Metalanguage Lessons high in metalanguage have high levels of talk about language and how texts work.	**Students' Self-Regulation** High student self-regulation is evident when the lesson proceeds without interruption and when students demonstrate autonomy and initiative in relation to their own behaviour in ways that allow the class to get on with learning.	**Connectedness** High connectedness is evident when learning has value and meaning beyond the classroom and school.
Substantive Communication Classes high in substantive communication have sustained interaction, communication focused on the substance of the lessons and reciprocal interaction.	**Student Direction** Classrooms with high student direction see students exercising control over one or more of the following aspects of a lesson: choice of activities, time spent on activities, pace of the lesson or criteria by which they will be assessed.	**Narrative** Use of narrative is high when the stories written, told, read, viewed or listened to help illustrate or bring to life the knowledge that students are addressing in the classroom.

NSW Department of Education, 2024



NT Government, Education Strategy, 2021



TAS Government, 2020

Focus areas	Schools are required to:
Data	› document, monitor and review attendance data[6], anecdotal evidence and assessment data, including learning across the phases of the *Continua of learning and development*, to determine focus areas for the sustained improvement of children's[7] learning, development and wellbeing › use a range of data together with the National Quality Standard (NQS) to inform quality improvement strategies for the kindergarten program and school performance planning › record and retain[8] data and documentation[9]
Curriculum	› enact the five learning and development areas of the *Queensland kindergarten learning guideline 2024* (QKLG) through planned and spontaneous child-centred experiences
Documenting, assessing and planning	› use an ongoing, strengths-based[10] planning process that includes: › gathering evidence and documenting learning › analysing and evaluating evidence across all learning and development areas using the *Continua of learning and development* › planning responsively for and from learning › implementing intentional learning experiences › critically reflecting[11] on teaching and learning › develop transition statements at the end of the year by collaborating with families, and reflecting on a range of documentation and evidence across the
Pedagogy	› use *A whole school approach to pedagogy* › employ a range and balance of age-appropriate, intentional and effective pedagogies outlined in the QKLG, including play-based and inquiry learning › explore and enhance digital learning in teaching and learning
Inclusion and diversity	› use the QKLG to support inclusive and equitable access, participation and engagement in kindergarten
Health and wellbeing education	› use the QKLG to build respectful and reciprocal relationships and support children's wellbeing
Managing risks in school curriculum activities	› provide risk assessment documentation, in accordance with the *SDK delivery procedure* and *Managing risks in school curriculum activities procedure*

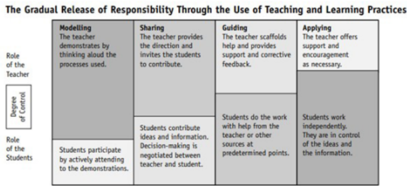
QLD Government, Curriculum, Assessment and Reporting Framework, 2024

How we teach in ACT public schools

Our schools are diverse, and our teachers are empowered learning professionals who use a variety of strategies to meet the individual needs of all students. We use student data to identify what each student needs, and we adjust our learning and teaching strategies accordingly.

In ACT public schools, we use the Gradual Release of Responsibility Model. This is a teaching strategy where the responsibility gradually shifts from the teacher to the student. The teacher models skills and concepts and transfers ownership of learning to the student to show their independence and understanding.

Teachers use a range of effective teaching and learning practices, allowing them to select the most suitable learning experiences. For example, strategies known as modelling, sharing and guiding are commonly used to teach students. Depending on what students need, a teacher might use modelling, where the teacher takes a higher degree of control to demonstrate new learning. If students are ready, a teacher will use guiding to create opportunities for students to learn new skills or concepts with support.

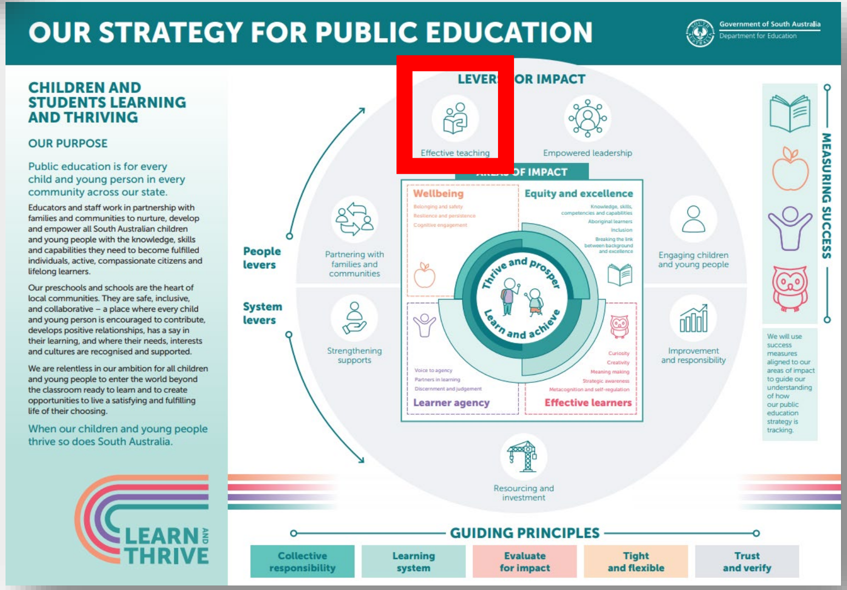


First Steps, *Linking Assessment, Teaching and Learning*. Department of Education WA, 2013.

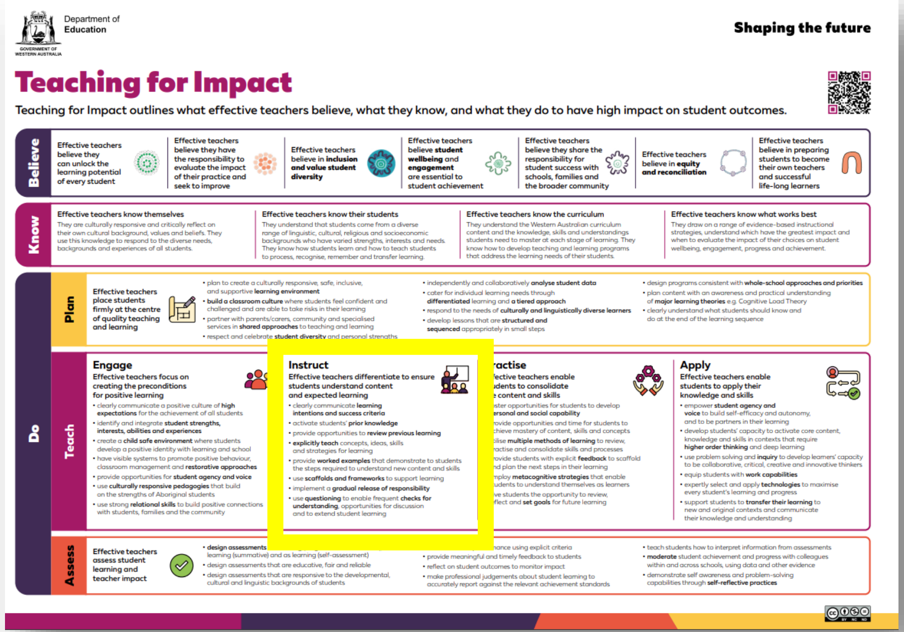
- **Explicit instruction:** This is a clear and direct way of teaching students new skills or concepts. The teacher breaks down the material into small steps and provides clear explanations and examples.
- **Differentiation:** This is the practice of adjusting our learning and teaching strategies to meet the individual needs of each student. It might involve adjusting the pace of instruction, the level of difficulty, or the type of learning experience.
- **Inquiry-based learning:** This is an active learning approach that encourages students to ask questions, think critically, and solve problems. Combining explicit and inquiry-based approaches gives students the foundational knowledge, concepts, and problem-solving strategies to explore and discover through their learning.
- **Personalised learning:** This is a process of tailoring the learning experience to each student's unique needs. This might involve providing additional support, using different learning materials, or offering more flexibility.

ACT Government, How we teach in ACT public schools, n.d.

ACU
AUSTRALIAN CATHOLIC UNIVERSITY



SA Government, Learn and Thrive, 2022



WA Government, Teaching for Impact, 2024

What do we know about students' mathematical reasoning?

Five ways to guide our thinking...

1

Students progressing in trajectories and in turn displaying mathematical thinking

(Bobis et al., 2005; Clements & Sarama, 2014; DiSessa, 2000; Empson, 2011; Fraivillig et al., 1999; Siemon et al., 2017; Simon & Tzur, 2004; Wilson et al., 2013).

2

Students engaging in problem solving tasks

(Boaler, 1998; Cheeseman, 2008; Fennema et al., 1998; Franke & Carey, 1997; Freiman, 2018; Mueller et al., 2014; Schoenfeld, 1987, 1988; van Bommel & Palmer, 2018; Wood et al., 2006).

3

Students engaging with strategies that support sense making

(Ball, 1993; Fennema et al., 1998; Ginsburg & Seo, 1999; Papandreou & Tsiouli, 2020; Steinberg et al., 2004; Wood et al., 2006).

4

Students displaying reasoning and justifying during mathematical learning experiences

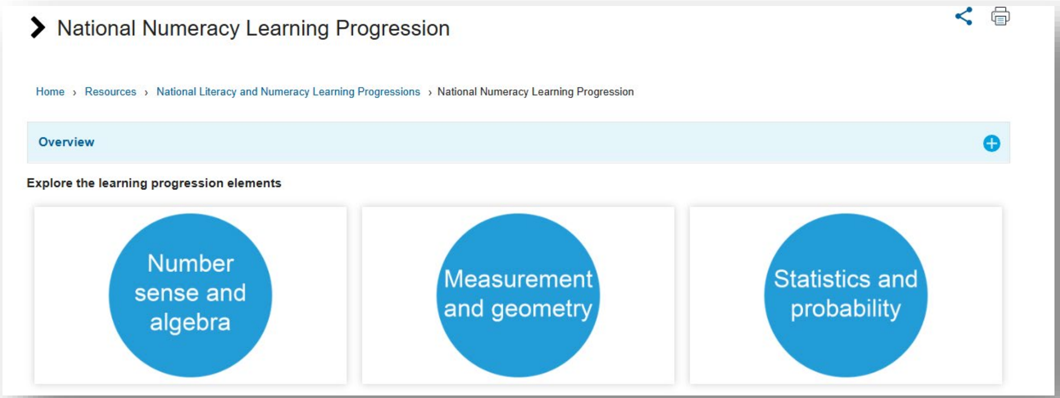
(Anthony et al., 2015; Carpenter et al., 2003; Diezmann et al., 2001; Henningsen & Stein, 1997; Herbert et al., 2015; Hunter & Anthony, 2011; Melhuish et al., 2020; Papic et al., 2009; Vale et al., 2017; Warren et al., 2013; Watters & English, 1995; Wood & McNeal, 2003).

5

Students making connections to known mathematical ideas and transferring their thinking

(Carpenter et al., 1990; Clements & Sarama, 2007; Fraivillig et al., 1999; Kinnear et al., 2018; Mulligan et al., 2006; Papic et al., 2011; Stein et al., 1996; Warren & Cooper, 2005; White, 1998).

Students progressing in trajectories and in turn displaying mathematical thinking



1

- Developmental progressions in learning mathematics concepts are referred to as learning trajectories, stepping stones, or growth points

- Learning trajectories help teachers gauge students' conceptual development and determine what learning needs to be addressed for the next task

- Learning trajectories, also known as learning sequences, guide teachers in understanding students' learning journeys and planning appropriate learning experiences

- Learning trajectories are multidimensional, encompassing mathematics goals, models of students’s thinking, instructional tasks, and their interaction

- Learning trajectories consist of three main components: a goal, a developmental progression, and instructional activities, with mathematical thinking occurring during the developmental progression

Students engaging in problem solving tasks

2

- The dispositions required to problem solve include:
- (a) preserving;
- (b) focusing attention on the problem;
- (c) testing the hypothesis;
- (d) taking risks;
- (e) displaying flexibility;
- (f) attempting to solve the problem in a different way; and,
- (g) displaying to self-regulate

- Students engage in problem solving by manipulating "physical objects, progress to various counting strategies, and then move on to more abstract strategies such as derived facts or invented algorithms"

- Students learn the processes of problem solving and the mathematical ideas that underpin that learning through exploring mathematical problems

Students engaging with strategies that support sense making

3

- Students benefit from engaging in multiple strategies to understand mathematics

- Invented strategies, such as solving problems without paper or pencil or using unconventional methods, aid in students' understanding of mathematics

- Inventing mathematical strategies lays the groundwork for tackling more complex mathematical concepts as students' progress

- Students' ability to invent strategies is evident across different age groups and is linked to deeper mathematical thinking

- Classroom environments that encourage inquiry and argumentation provide opportunities for students to express and develop their invented methods

Sense-Making | Adding Two-Digit Numbers

What is 24 + 58?

Sense-Making | Adding Two-Digit Numbers

What is
24 + 58?

Bill:	[*Writing out the traditional addition algorithm*] 8 + 4 is 12. Write the 2 here and the 1 up here. 1 + 2 + 4 = 7. Write the 7 next to the 2.

$$\begin{array}{r} {\scriptstyle 1} \\ 24 \\ +48 \\ \hline 72 \end{array}$$

Teacher:	Is this really a 1?
Bill:	Yes, it came from the 12; you're not allowed to put it next to the 2.
Teacher:	Why not?
Bill:	You're just not allowed to do that.

Battista, 2018

Sense-Making | Adding Two-Digit Numbers

Teacher: All year we have been talking about addition, and you have invented a bunch of ways to add two-digit numbers. Today, I want us to take a look at all these ways. I'd like you each to solve this problem in several different ways. Then, we'll talk about what you did.

Teacher: [*After students worked on the problem 24 + 48 by themselves at their seats*] Okay, I want to hear what kinds of ways you used to solve this problem, and I want you explain to the class why you did what you did.

Fred: I remember doing the problem like this: 20, 30, 40, 50, 60, 70, 72.

Teacher: Explain what you did.

Fred: I started with the 20 in 24 and counted tens in 48. Then I counted 10 more from the 12, then 2 more.

Teacher: Where did you get 12?

Fred: It's just 4 + 8 = 12. From the 24 and 48.

Teacher: Okay, how about a different way?

Mary: I did 48, 58, 68, 70, 72.

Teacher: Explain what you did.

Mary: I started with 48, and then I did the 24. I counted 10 onto 48, then 10 more, and then 2 + 2 makes 24. If you do one part at a time, it's easy.

Teacher: Okay, how about another way?

Jon: I did 40 + 20 = 60; 8 + 4 = 12; 60 + 12 = 72.

Teacher: How did you know you could do that?

Jon: You just add the tens, then add the ones, then add them together.

Teacher: Okay, how about another different way?

Serena: I need to write it. [*Goes to board and writes as shown below, then explains*] Here's how I added: 8 + 4 is 12. So I wrote a 2 from 12 under the 8 and put the 10 from 12 over the 20, because it's tens. Then I added 10 and 20 and 40, and I got 70. And 70 + 2 is 72.

$$\begin{array}{r@{}l} & \scriptstyle 10 \\ & 20 + 4 \\ + & \underline{40 + 8} \\ & 70 + 2 = 72 \end{array}$$

Teacher: Why did you write the 2 under the 8?

Serena: You just put the 2 ones from 12 in the ones place that's under the 4 and 8.

Battista, 2018

Students displaying reasoning and justifying during mathematical learning experiences

Mathematical reasoning encompasses various types including inductive, deductive, abductive, and adaptive reasoning:

4

- Inductive reasoning: verifying predictions

- Abductive reasoning: find the best explanation with the use of additional information to build on new learning

- Deductive reasoning: making predictions

- Adaptive reasoning: using a collection of findings, processes, concepts and solutions, to make sense of the mathematical problems

- Justifying solutions is a key aspect of mathematical reasoning

- Encouraging student discussion and explanation fosters the development of reasoning and justifying skills

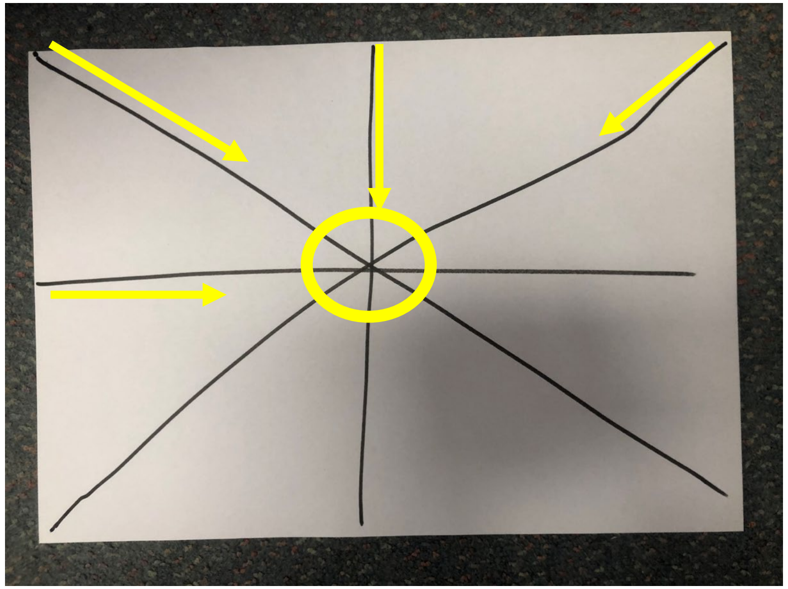
- Students' ability to articulate their mathematical thinking improves through activities that promote mathematical inquiry and scaffolded participation over time

Reasoning | Finding the Middle

Learning experience one	Physical manipulatives		Example teacher questions
Framed photograph—finding the middle This is a framed photograph of my son Joey. (*Hold up real framed photograph*.) I have a blank wall at home, and I would like to hang this photograph in the middle of that wall. Let's imagine this A3 piece of paper (hold up A3 paper) is the blank wall and this is a smaller picture frame (hold up small picture frame). Open-ended question: How can you find the exact place to hang up Joey's photograph?	 Photo frame (measures 20 cm × 15 cm	 Smaller laminated photo frame (measures 4 cmx x cm)	How can we check that's exactly the middle? What can we do to check? Is there a way that we can find exactly the middle of the entire wall?

Monteleone, 2023

Reasoning | Finding the Middle



Student

- Are you going to give me a ruler?

Teacher

- No

Student

- You can’t fold a wall so you can’t fold this paper.
- *I will draw a line here and another line, here and just to prove it to you I will draw another line this way and another line this way, that is the middle.*

Monteleone, 2023

Students displaying reasoning and justifying during mathematical learning experiences

What type of reasoning was this student displaying?

4

Mathematical reasoning encompasses various types including inductive, deductive, abductive, and adaptive reasoning:

- Inductive reasoning: verifying predictions

- Abductive reasoning: find the best explanation with the use of additional information to build on new learning

- Deductive reasoning: making predictions

- Adaptive reasoning: using a collection of findings, processes, concepts and solutions, to make sense of the mathematical problems

- Justifying solutions is a key aspect of mathematical reasoning

- Encouraging student discussion and explanation fosters the development of reasoning and justifying skills

- Students' ability to articulate their mathematical thinking improves through activities that promote mathematical inquiry and scaffolded participation over time

Students making connections to known mathematical ideas and transferring their thinking

ACU
AUSTRALIAN CATHOLIC UNIVERSITY

5

- Recognising interrelationships within mathematical concepts is a crucial aspect of reasoning and mathematical thinking

- Making mathematical connections involves identifying links within and between mathematical ideas, which indicates high levels of mathematical thinking

- Purposefully constructed learning experiences that build on prior knowledge and make conceptual connections engage students in higher levels of mathematical thinking

Making Connections | How many tiles?

Learning Experience	Physical manipulatives		Example teacher questions
Cubby house – identifying number of tiles required I have just finished building a cubby house for my children at home (*show picture of the cubby house*). I would like to put these tiles down on the floor of the cubby house (*show square tile*). Open-ended question: How can I work out how many tiles I need?	 Image of a cubby house	 Square tile (measures 5 cm x 5 cm)	So how do you know? How can I figure out how many tiles I need to put on the floor of the cubbyhouse?

Monteleone, 2023

Making Connections | How many tiles?

Student

- You could *get a measuring tape and then pull it across like that, where the corners are.*
- Then you'll get five tiles, five tiles, five tiles. You keep getting five tiles and then you start laying them out in a row of five, like this.
- One, two, three, four, five.

Teacher

- Do you think that only five tiles can fit across here? Is that what you think?

Student

- Well maybe you could use wood to do it.

Teacher

- Wood? Explain how you could use wood.

Student

- Pretend this is a whole piece of wood.
- And this is a whole piece of wood.
- It would go across the cubby house.
- And this is a piece of wood. And then that's a piece of wood.
- This is a piece of wood.
- And those last other ones could go, these last other ones, could go on top here.
- And one will go here.
- *And so, what you need to buy is one more tile.*

Monteleone, 2023

Five ways to guide our thinking…

1

Students progressing in trajectories and in turn displaying mathematical thinking

(Bobis et al., 2005; Clements & Sarama, 2014; DiSessa, 2000; Empson, 2011; Fraivillig et al., 1999; Siemon et al., 2017; Simon & Tzur, 2004; Wilson et al., 2013).

2

Students engaging in problem solving tasks

(Boaler, 1998; Cheeseman, 2008; Fennema et al., 1998; Franke & Carey, 1997; Freiman, 2018; Mueller et al., 2014; Schoenfeld, 1987, 1988; van Bommel & Palmer, 2018; Wood et al., 2006).

3

Students engaging with strategies that support sense making

(Ball, 1993; Fennema et al., 1998; Ginsburg & Seo, 1999; Papandreou & Tsiouli, 2020; Steinberg et al., 2004; Wood et al., 2006).

4

Students displaying reasoning and justifying during mathematical learning experiences

(Anthony et al., 2015; Carpenter et al., 2003; Diezmann et al., 2001; Henningsen & Stein, 1997; Herbert et al., 2015; Hunter & Anthony, 2011; Melhuish et al., 2020; Papic et al., 2009; Vale et al., 2017; Warren et al., 2013; Watters & English, 1995; Wood & McNeal, 2003).

5

Students making connections to known mathematical ideas and transferring their thinking

(Carpenter et al., 1990; Clements & Sarama, 2007; Fraivillig et al., 1999; Kinnear et al., 2018; Mulligan et al., 2006; Papic et al., 2011; Stein et al., 1996; Warren & Cooper, 2005; White, 1998).

Critical Thinking (CT)

+

Mathematical Thinking (MT)

→

Critical Mathematical Thinking (CMT)

Critical Mathematical Thinking For Young Students Conceptual Framework

Interpreting

- *Clarifying*
- *Estimating*

Analysing

- *Noting relationships*
- *Applying*
- *Querying*

Evaluating

- *Solving*
- *Offering opinions and reasons*
- *Assessing claims and arguments*
- *Making judgements with criteria*

Explaining

- *Justifying*
- *Stating*
- *Presenting*

Creating

- *Non-algorithmic decision making*
- *Self-regulating*

Monteleone, 2022; Monteleone et al., 2023

The five CMT themes

Interpreting	*Analysing*	*Evaluating*	*Explaining*	*Creating*
• Is an essential part of critical thinking, as it involves the formation of logical judgments or conclusions. • Critical thinkers who make decisions may also engage in interpretation. • The literature on interpreting that is part of the CMTFYS include clarifying and estimating.	• Is recognised as an important component of critical thinking. • Is both a cognitive skill and an affective disposition. • The sub-themes that represent analysing in critical thinking include applying, questioning, and noting relationships.	• Making claims and thought processes has been identified as an essential practice for promoting mathematical thinking. • The literature on mathematical thinking also identifies sub-themes of evaluating, including making judgments based on criteria, solving problems, and providing opinions supported by reasoning.	• *This occurs* when an individual provides reasons for decisions made, as well as depth and detail of the explanation. • The sub-themes, such as stating, presenting, and justifying, help individuals develop their critical thinking skills by enabling them to explain their thought processes and how they arrived at their judgments.	• This involves generating new and innovative ideas. • Sub-themes associated with creating and critical thinking are related to evaluation and decision-making. • One such sub-theme is self-regulation, which involves an individual's ability to evaluate their own inferences. Non-algorithmic decision-making, which involves mental processes, strategies, and representations that people use to solve problems and make decisions, is also as a critical thinking element.

Victorian Teaching and Learning Model 2.0

ACU AUSTRALIAN CATHOLIC UNIVERSITY

Elements of learning

Attention, focus and regulation	Knowledge and memory	Retention and recall	Mastery and application
Refers to learning requiring students' attention and involving active engagement in a supportive and responsive learning-focused environment.	Refers to students processing new information in their working memory, where they connect it with existing knowledge in long-term memory, building mental models that integrate and organise knowledge.	Refers to working memory being able to hold a small amount of information at once (cognitive load). If overloaded, new knowledge won't be effectively stored in long-term memory.	Refers to consistent practice and retrieval, allowing students to develop and demonstrate mastery by retaining knowledge and understanding how to apply it effectively.

Interpreting

Explaining

Evaluating

Creating

Analysing

How does CMT link to the Early Years Learning Framework (EYLF)?

- In the elaborations of the outcomes within the EYLF, mathematical thinking may occur when children problem solve. For example, one outcome from the EYLF, Children and Confident and Involved Learners, highlights mathematical thinking opportunities, as follows:

create and use representation to organise, record and communicate mathematical ideas and concepts;

make predictions and generalisations about their daily activities, aspects of the natural world and environments, using patterns they generate or identify and communicate these using mathematical language and symbols;

contribute to mathematical discussions and arguments.

(Australian Government Department of Education, p. 53, 2022)

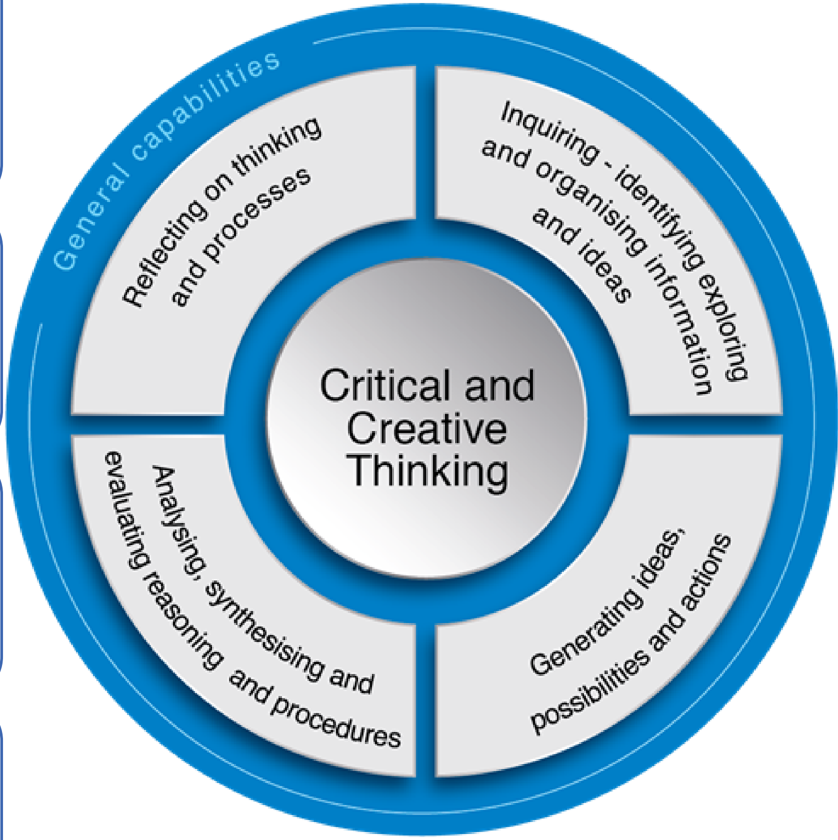
How does CMT link to the Australian Curriculum?

For primary aged children, the Australian Curriculum (2022) includes general capabilities that teachers are to address across all subject areas, including mathematics.

One general capability that aligns with critical mathematical thinking is *Critical and Creative Thinking.*

Critical thinking involves recognising arguments, evaluating evidence, and solving problems, while creative thinking involves generating new ideas and finding innovative solutions.

In the Australian curriculum for Mathematics F – 6, the four proficiencies help enhance CMT. They include understanding, fluency, problem-solving and reasoning.



Critical and Creative Thinking (Version 8.4)

Reasoning in the curriculum

F-2

- Increasingly sophisticated capacity for logical thought and actions.
- Reasoning occurs when they **explain their thinking**.

3 – 6

- Increasingly sophisticated capacity for logical thought and actions, such as evaluating, explaining and generalising.
- Reasoning occurs when they **explain their thinking, adapt the known to the unknown, and transfer learning from one context to another and explain their choices**.

7 - 8

- Increasingly sophisticated capacity for logical thought and actions, such as analysing, evaluating, explaining, justifying and generalising.
- Reasoning occurs when they **explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, compare and contrast related ideas and explain their choices**.

9 – 10

- Increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising.
- Reasoning occurs when **they explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, prove that something is true or false, compare and contrast related ideas and explain their choices.**

Australian Curriculum. Mathematics Proficiencies, n.d.

Mathematics scope and sequence – Foundation to Level 6

2.0

Mathematics scope and sequence – Foundation to Level 6

Strand: Number

Foundation	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Content descriptions						
name, represent and order numbers, including zero to at least 20, using physical and virtual materials and numerals VC2MFN01	recognise, represent and order numbers to at least 120 using physical and virtual materials, numerals, number lines and charts VC2M1N01	recognise, represent and order numbers to at least 1000 using physical and virtual materials, numerals and number lines VC2M2N01	identify, explain and use the properties of odd and even numbers VC2M3N01	recognise and extend the application of place value to tenths and hundredths and use the conventions of decimal notation to name and represent decimals VC2M4N01	interpret, compare and order numbers with more than 2 decimal places, including numbers greater than one, using place value understanding; represent these on a number line VC2M5N01	recognise situations, including financial contexts, that use integers; locate and represent integers on a number line and as coordinates on the Cartesian plane VC2M6N01
recognise and name the number of objects within a collection up to 5 using subitising VC2MFN02			recognise, represent and order natural numbers using naming and writing conventions for numerals beyond 10 000 VC2M3N02	investigate number sequences involving multiples of 3, 4, 6, 7, 8 and 9 VC2M4N02	express natural numbers as products of their factors, recognise multiples and determine if one number is divisible by another VC2M5N02	identify and describe the properties of prime, composite, square and triangular numbers and use these properties to solve problems and simplify calculations VC2M6N02
quantify and compare collections to at least 20 using counting and explain or demonstrate reasoning VC2MFN03						
	partition one- and two-digit numbers in different ways using physical and virtual materials, including partitioning two-digit numbers into tens and ones VC2M1N02	partition, rearrange, regroup and rename two- and three-digit numbers using standard and non-standard groupings; recognise the role of a zero digit in place value notation VC2M2N02				
	quantify sets of objects, to at least 120, by partitioning collections into equal groups using number knowledge and skip counting VC2M1N03					
		recognise and describe one-half as one of 2 equal parts of a whole and connect halves, quarters and eighths through repeated halving VC2M2N03	recognise and represent unit fractions including $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{10}$ and their multiples in different ways; combine fractions with the same denominator to complete the whole VC2M3N03	find equivalent representations of fractions using related denominators and make connections between fractions and decimal notation VC2M4N03	compare and order common unit fractions with the same and related denominators, including mixed numerals, applying knowledge of factors and multiples; represent these fractions on a number line VC2M5N03	apply knowledge of equivalence to compare, order and represent common fractions, including halves, thirds and quarters, on the same number line and justify their order VC2M6N03

[Mathematics V2 scope and sequence Levels F–6.docx]

Reasoning	Reason*
𝍸 𝍸 |	𝍸 𝍸

quantify and compare collections to at least 20 using counting and explain or demonstrate reasoning

VC2MFN03

add and subtract one- and two-digit numbers, represent problems using number sentences and solve using part-part-whole reasoning and a variety of calculation strategies

VC2M2N04

choose and use estimation and rounding to check and explain the reasonableness of calculations, including the results of financial transactions

VC2M4N07

Mathematics scope and sequence – Levels 7 – 10A

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

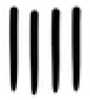
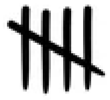
VICTORIA State Government

Mathematics scope and sequence – Levels 7–10A

Strand: Number

Level 7	Level 8	Level 9	Level 10	Level 10A
Content descriptions				
describe the relationship between perfect square numbers and square roots, and use squares of numbers and square roots of perfect square numbers to solve problems VC2M7N01	recognise irrational numbers in applied contexts, including π and numbers that develop from the square root of positive real numbers that are not perfect squares, and recognise that irrational numbers cannot develop from the division of integer values by natural numbers VC2M8N01	recognise that the real number system includes the rational numbers and the irrational numbers, and solve problems involving real numbers with and without using digital tools VC2M9N01		use the definition of a logarithm to establish and apply the laws of logarithms and investigate logarithmic scales in measurement VC2M10AN03
represent natural numbers in expanded notation using powers of 10, and as products of powers of prime numbers using exponent notation VC2M7N02	establish and apply the exponent laws with integer exponents and the zero exponent, using exponent notation with numbers VC2M8N02			
find equivalent representations of rational numbers and represent positive and negative rational numbers and mixed numbers on a number line VC2M7N03	convert between fractions and terminating or recurring decimals, using digital tools as appropriate VC2M8N03			
round decimals to a given accuracy appropriate to the context and use appropriate rounding and estimation to check the reasonableness of computations VC2M7N04				
multiply and divide fractions and decimals using efficient mental and written strategies, and digital tools VC2M7N05	use the 4 operations with integers and with rational numbers, choosing and using efficient mental and written strategies, and digital tools where appropriate, and making estimates for these computations VC2M8N04		recognise the effect of using approximations of real numbers in repeated calculations and compare the results when using exact representations VC2M10N01	define rational and irrational numbers and perform operations with surds and fractional indices VC2M10AN01
use the 4 operations with positive rational numbers, including fractions and decimals, to solve problems using efficient mental and written calculation strategies VC2M7N06	solve problems involving the use of percentages, including percentage increases and decreases and percentage error, with and without digital tools VC2M8N05			perform operations on numbers involving fractional exponents and surds VC2M10AN02
find percentages of quantities and express one quantity as a percentage of another, with and without digital tools VC2M7N07				
compare, order and solve problems involving addition and subtraction of integers VC2M7N08				
recognise, represent and solve problems involving ratios VC2M7N09				

https://victoriancurriculum.vcaa.vic.edu.au/mathematics/mathematics-version-2-0/introduction/scope-and-sequence

Reasoning	Reason*
	

round decimals to a given accuracy appropriate to the context and use appropriate rounding and estimation to check the reasonableness of computations VC2M7N04

establish properties of quadrilaterals using congruent triangles and angle properties, and solve related problems explaining reasoning VC2M8SP02

Teacher questioning

Teacher questioning – in a primary mathematics classroom setting

75% Teacher Questions = Yes or No Answers

Factual questions

Probing questions

Guiding questions

Factual Teacher Questions

Factual questions tend to provide very little information about a student's understanding of a concept or content.

These questions are lower order with little opportunity to discuss strategies with others (Sahin & Kulm, 2008).

What is the name of figure X?

Which figure has five sides?

How many figures have an acute angle?

Is 76 larger than 67?

Probing Teacher Questions

Probing questions have been found to extend student's understanding, knowledge, and mathematical thinking.

These questions move students from low level to higher order thinking (Sahin & Kulm, 2008) providing further clarity about a student's explanation.

Is there another way to solve this?

What do you mean by X?

How did you work that out; do you think it matters?)

What would happen if…?

Okay, why?

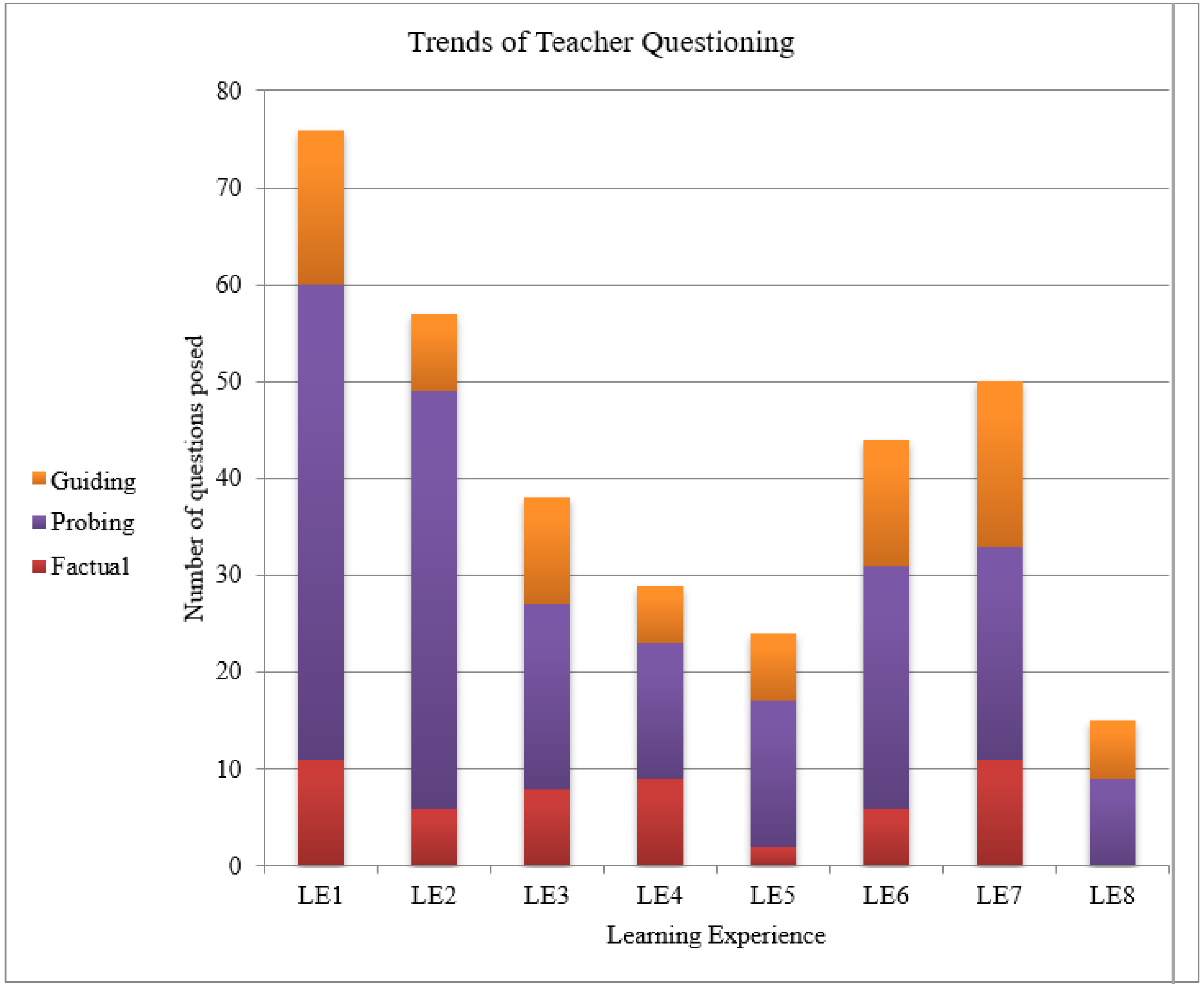
Guiding Teacher Questions

Guiding questions are considered questions that direct students to derive concepts or procedures to solve problems (Mata-Pereira & da Ponte, 2017).

These questions are used when the teacher is supporting students to discuss their solutions, strategies, or procedures they used in the learning experience.

Which method do you need to use now?	Have you developed a strategy to …?
Are there any other options…?	What do you notice about this problem?
What facts does the problem tell you?	What are you trying to find out? (What is the problem asking for?
What tool might help you?	How could drawing, table or diagram help you?

Analysis of my teacher questioning



Summary of the Types of Teacher Questions

Factual Questions	Probing Questions	Guiding Questions
• Provide very little information about a student's understanding of a concept or content • Are lower order with little opportunity to discuss strategies with others	• Can extend student's understanding, knowledge, and mathematical thinking • Support students to move from low level to higher order thinking • Provide further clarity about a student's explanation	• Direct students to derive concepts or procedures to solve problems • Can support students to discuss their solutions, strategies, or procedures they used in the learning experience
What is the name of figure X? Which figure has five sides? How many figures have an acute angle? Is 76 larger than 67?	Is there another way to solve this? What do you mean by X? How did you work that out; do you think it matters?) What would happen if…? Okay, why?	Which method do you need to use now? Have you developed a strategy to …? Are there any other options…? What do you notice about this problem? What facts does the problem tell you? What are you trying to find out? (What is the problem asking for? What tool might help you? How could drawing, table or diagram help you?

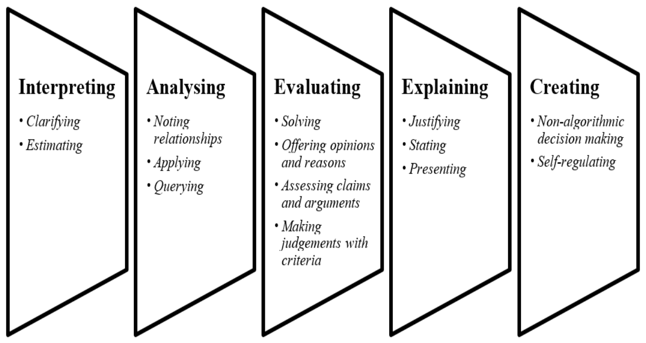
Why do we ask these questions – enhancing Reasoning and Critical Mathematical Thinking

Probing questions were predominantly used to support students in eliciting their CMT.

Guiding and factual questions were less prevalent in the interviews.

Different types of questioning were analysed in relation to the CMTFYS.

Specific question types can aid students in interpreting and explaining their CMT during mathematical learning experiences



What type of learning is best situated for CMT?

Students use mathematical proof to justify solutions

Students of primary and middle-school age can articulate arguments

Students develop justifications through making sense of problems, noticing patterns, and posing hypotheses

Discussions and negotiations with peers help students refine their solutions and understanding of mathematical reasoning

When students articulate convincing justifications, they further refine their understanding and ability to validate mathematical statements

Was this presentation about explicit teaching in mathematics?

ACU
AUSTRALIAN CATHOLIC UNIVERSITY

How a more explicit teaching style helped this school defy NAPLAN struggles elsewhere

By national education and parenting reporter Conor Duffy and the Specialist Reporting Team's Evan Young
Posted Thu 24 Aug 2023 at 4:59am

Nasya Hassan is a teacher at Sydney Adventist School, where NAPLAN results have seen a big turnaround. *(ABC News: Marcus Stimson)*

THE EDUCATION STATE | VICTORIA State Government | Department of Education

Victorian Teaching and Learning Model 2.0

Elements of learning

Attention, focus and regulation	Knowledge and memory	Retention and recall	Mastery and application
Refers to learning requiring students' attention and involving active engagement in a supportive and responsive learning-focused environment.	Refers to students processing new information in their working memory, where they connect it with existing knowledge in long-term memory, building mental models that integrate and organise knowledge.	Refers to working memory being able to hold a small amount of information at once (cognitive load). If overloaded, new knowledge won't be effectively stored in long-term memory.	Refers to consistent practice and retrieval, allowing students to develop and demonstrate mastery by retaining knowledge and understanding how to apply it effectively.

Elements of teaching

Planning	Refers to the collaborative development of whole school teaching and learning programs that break down and sequence the knowledge to be taught and assessed. It also refers to the planning required to implement the curriculum into the classroom and to the school-wide enactment of a multi-tiered system of supports.
Enabling learning	Refers to the positive relationships, cultural responsiveness, classroom expectations and management techniques that teachers establish and use to foster student self-regulation and self-efficacy, and to create a learning-focused environment where the development and application of knowledge drives curiosity and creativity.
Explicit teaching	Refers to the evidence-based practices that manage the cognitive load of students, including activating prior knowledge, clearly stating learning objectives, providing explicit explanations of new knowledge, scaffolding learning and modelling practice, and using formative assessment and feedback to monitor progress towards mastery.
Supported application	Refers to the practices that maximise the consolidation and application of learning, including revisiting and reviewing knowledge, varying and spacing practice, organising knowledge and extending and challenging students as they move to mastery of new factual, conceptual and procedural knowledge.

* Knowledge includes factual, conceptual and procedural knowledge. See AERO (2023), https://www.edresearch.edu.au/resources/how-students-learn-best, p.5

This was published 6 months ago

Education boss calls for doubling down on explicit teaching in schools

Lucy Carroll
October 23, 2023 – 5.00am

Save | Share | A A A | 76 View all comments

Listen to this article
6 min

The head of the NSW public education system has called on schools to double down on the use of explicit instruction – a teaching method that gives students step-by-step and clear instructions – in a bid to boost results and close the stark achievement gap.

How an explicit teaching approach optimises learning

Breaking down and fully explaining content helps students transfer information to memory

Providing opportunities for review and practise helps students retain and recall what they learn

Organising and sequencing content around a specific objective deepens student understanding

Knowledge-building, guidance and scaffolding supports students with additional limitations in prior knowledge, memory and information processing

A strong foundation of knowledge and skills provides students with mental models for extending and applying their learning with greater independence

Key take-aways

Read about Critical Mathematical Thinking

Monteleone, C., Miller, J., & Warren, E. (2023). Conceptualising critical mathematical thinking in young students. Mathematics Education Research Journal, 35(2), 339-359.

Monteleone, C., & Miller, J. (2023). Evidence of Young Students' Critical Mathematical Thinking. Mathematics Education Research Group of Australasia.

Monteleone, C. (2022). Teacher Questioning to Support Young Students to Interpret and Explain Their Critical Mathematical Thinking. Mathematics Education Research Group of Australasia.

Pose guiding and probing questions

Rethink your explicit teaching strategies

What is best in mathematics deserves not merely to be learned as a task but to be assimilated as a part of daily thought and brought again and again before the mind with ever-renewed encouragement.

Bertrand Rusell, in his book *Contemplation and Action, 1902-14* (1985), stressing the continuous interplay of reasoning and reflection in mathematics.

Reasoning is the lifeblood of mathematics;
the air it breathes.

Hannah Fry is a British academic, author and radio and television presenter. She is Professor in the Mathematics of Cities at the UCL Centre for Advanced Spatial Analysis.

Reference List

ACT Government. (n.d.). How we teach in ACT public schools. https://www.education.act.gov.au/public-school-life/Our-Curriculum/teaching-and-learning-in-act-public-schools
Anthony, G., Hunter, J., & Hunter, R. (2015). Supporting prospective teachers to notice students' mathematical thinking through rehearsal activities. *Mathematics Teacher Education and Development, 17(2)*, 7-24.
Australian Curriculum (n.d.). Meeting the needs of gifted and talented students. https://www.australiancurriculum.edu.au/resources/student-diversity/meeting-the-needs-of-gifted-and-talented-students/
Australian Curriculum. Mathematics Proficiencies, n.d. https://www.australiancurriculum.edu.au/resources/mathematics-proficiencies/portfolios/reasoning/
Australian Government Department of Education. (2022). Belonging, Being and Becoming. The Early Years Learning Framework.
Australian Education Research Organisation. (2023).Explicit Instruction. https://www.edresearch.edu.au/sites/default/files/2023-11/explicit-instruction-optimises-learning-aa.pdf
Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. Journal of Teacher Education, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554
Batista, M. (2018(). Ressoning and sense making in the mathematics classroom: Grades 6-8. National Council of Teachers of Mathematics
Boaler, J. (1998). Open and closed mathematics: Student Experiences and understandings. *Journal for Research in Mathematics Education*, *29*(1), 41–62. https://doi.org/10.2307/749717
Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, B., Young-Loveridge, J., & Gould, P. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27-57. https://doi.org/10.1007/bf03217400
Carpenter, T. P., Megan Loef Franke, & Levi, L. (2003). *Thinking mathematically: integrating arithmetic and algebra in elementary school.* Heinemann.
Cheeseman, J. (2008). Young children recollect their mathematical thinking. *In Proceedings of the 32nd conference of the International Group for the Psychology of Mathematics Education* (*Vol. 2*, pp. 289-296).
Clements, D. H., & Sarama, J. (2007). Effects of a Preschool Mathematics Curriculum: Summative Research on the Building Blocks Project. *Journal for Research in Mathematics education, 38*(2), 136-163. https://doi.org/10.2307/30034954
Clements, D. H., & Sarama, J. (2014). Developing young children's mathematical thinking and understanding. *The Routledge International Handbook of Young Children's Thinking and Understanding*. Routledge: New York.
Diezmann, C.M., Watters, J. J., & English, L. D. (2001) Implementing mathematical investigations with young children. *In Proceedings 24th Annual Conference of the Mathematics Education Research Group of Australasia*, pages 170-177, Sydney.
DiSessa, A. (2000). *Changing minds: Computers, learning, and literacy*. Cambridge, MA: MIT Press.
Empson, S. B. (2011). On the idea of learning trajectories: Promises and pitfalls. *The Mathematics Enthusiast, 8*(3), 571-596.
Fennema, E., Carpenter, T. P., Jacobs, V. R., Franke, M. L., & Levi, L. W. (1998). A longitudinal study of gender differences in young children's mathematical thinking. *Educational researcher, 27*(5), 6-11.
Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in everyday mathematics Classrooms. *Journal for Research in Mathematics education, 30*(2), 148-170. https://doi.org/10.2307/749608
Franke, M. L., & Carey, D. A. (1997). Young children's perceptions of mathematics in problem-solving environments. *Journal for Research in Mathematics education, 28*(1), 8-25. https://doi.org/10.2307/749661
Freiman, V. (2018). Complex and open-ended tasks to enrich mathematical experiences of kindergarten students. In F. M. Singer (Ed.), *Mathematical Creativity and Mathematical Giftedness: Enhancing Creative Capacities in Mathematically Promising Students* (pp. 373-404). Cham: Springer International Publishing.
Ginsburg, H. P., & Seo, K.-H. (1999). Mathematics in children's thinking. *Mathematical thinking and learning, 1*(2), 113-129. https://doi.org/10.1207/s15327833mtl0102_2
Henningsen, M., & Stein, M. K. (1997). Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning. *Journal for Research in Mathematics education, 28*(5), 524-549.
Herbert, S., Vale, C., Bragg, L. A., Loong, E., & Widjaja, W. (2015). A framework for primary teachers' perceptions of mathematical reasoning. *International Journal of Educational Research, 74*, 26-37. https://doi.org/10.1016/j.ijer.2015.09.005
Hunter, R., & Anthony, G. (2011). Forging mathematical relationships in inquiry-based classrooms with Pasifika students. *Journal of Urban Mathematics Education, 4*(1), 98-119.

Reference List Continued

ACU
AUSTRALIAN CATHOLIC UNIVERSITY

Kinnear, V., Lai, M.Y., & Muir, T . (2018). Forging connections in early mathematics: perspectives and provocations. In *Forging Connections in Early Mathematics Teaching and Learning* (pp. 1-15). Springer, Singapore.

Mata-Pereira, J., & da Ponte, J.-P. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. Educational Studies in Mathematics, 96(2), 169-186. https://doi.org/10.1007/s10649-017-9773-4

Melhuish, K., Thanheiser, E., & Guyot, L. (2020). Elementary school teachers' noticing of essential mathematical reasoning forms: justification and generalization. *Journal of Mathematics Teacher Education, 23*(1), 35-67. https://doi.org/10.1007/s10857-018-9408-4

Monteleone, C. (2022). Teacher Questioning to Support Young Students to Interpret and Explain Their Critical Mathematical Thinking. Mathematics Education Research Group of Australasia.

Monteleone, C., White, P., & Geiger, V. (2018). Defining the Characteristics of Critical Mathematical Thinking. Mathematics Education Research Group of Australasia.

Mueller, M., Yankelewitz, D., & Maher, C. (2014). Teachers promoting student mathematical reasoning. *Investigations in Mathematics Learning, 7*(2), 1-20. https://doi.org/10.1080/24727466.2014.11790339

Mulligan, J., Papic, M., Prescott, A., & Mitchelmore, M. (2006). *Improving early numeracy through a pattern and structure mathematics awareness program (PASMAP).* Paper presented at the Conference of the Mathematics Education Research Group of Australasia.

Northen Territory Education Government. (2021). Education Strategy. https://education.nt.gov.au/__data/assets/pdf_file/0007/1061386/education-NT-strategy-2021-2025.pdf

NSW Department of Education (2024). Teaching and Learning. https://education.nsw.gov.au/teaching-and-learning/professional-learning/quality-teaching-rounds

Papandreou, M., & Tsiouli, M. (2020). Noticing and understanding children's everyday mathematics during play in early childhood classrooms. *International Journal of Early Years Education*, 1–18. https://doi.org/10.1080/09669760.2020.1742673

Papic, M., Mulligan, J., & Bobis, J. (2009). Developing Mathematical Concepts in Australia Pre-School Settings: Children's Mathematical Thinking. *Paper presented at the MERGA,* New Zealand.

Papic, M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3), 237. https://doi.org/10.5951/jresematheduc.42.3.0237

QLD Government (2024). Curriculum, Assessment and Reporting Framework. https://education.qld.gov.au/curriculums/Documents/k-12-curriculum-assessment-reporting-framework.pdf

South Australian Government. (2022). Learn and Thrive Strategic Plan. https://www.education.sa.gov.au/purpose-media/Public-Education-Strategic-Plan.pdf

Sahin, A., & Kulm, G. (2008). Sixth grade mathematics teachers' intentions and use of probing, guiding, and factual questions. Journal of Mathematics Teacher Education, 11(3), 221-241. https://doi.org/10.1007/s10857-008-9071-2Schoenfeld, A. H. (1987). Pólya, problem solving, and education. *Mathematics magazine, 60*(5), 283-291.

Siemon, D., Horne, M., Clements, D., Confrey, J., Maloney, A., Sarama, J.,Tzue, R., & Watson, A. (2017). *Researching and using learning progressions (trajectories) in mathematics education.* Paper presented at the 41st Conference of the International Group for the Psychology of Mathematics Education.

Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical thinking and learning, 6*(2), 91-104.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*(1), 50-80.

Steinberg, R. M., Empson, S. B., & Carpenter, T. P. (2004). Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education, 7*(3), 237-267.

TAS Government. (2020). A Pedagogical Framework. https://www.parliament.tas.gov.au/__data/assets/pdf_file/0027/59922/department20of20education-new2020brighton20high20school20-20attachment20720-20doe20pedagogical20framework.pdf

Vale, C., Widjaja, W., Herbert, S., Bragg, L. A., & Loong, E. Y.-K. (2017). Mapping variation in children's mathematical reasoning: the case of 'what else belongs?'. *International Journal of Science and Mathematics Education, 15*(5), 873-894.

van Bommel, J., & Palmér, H. (2018). Problem Solving in Early Mathematics Teaching—A Way to Promote Creativity?. *Creative Education, 9*(12), 1775-1793.

Victorian State Government. (2023). Victorian Teaching and Learning Model 2.0. https://content.sdp.education.vic.gov.au/media/victorian-teaching-and-learning-model-2-0-2816

Warren, E., & Cooper, T. J. (2005). Young children's ability to use the balance strategy to solve for unknowns. *Mathematics Education Research Journal, 17*(1), 58-72. https://doi.org/10.1007/bf03217409

Warren, E., Miller, J., & Cooper, T. J. (2013). Exploring young students' functional thinking. *PNA, 7*(2), 75-84.

Watters, J. J., & English, L. D. (1995). Children's application of simultaneous and successive processing in inductive and deductive reasoning problems: Implications for developing scientific reasoning skills. *Journal of Research in Science Teaching, 32*(7), 699-714.

WA Government (2024). Teaching for Impact. https://www.education.wa.edu.au/quality-teaching-strategy

White, C. S. (1998). The relationship between young children's analogical reasoning and mathematical learning. *Mathematical Cognition, 4*(2), 103-123. https://doi.org/10.1080/135467998387352

Wood, T., & McNeal, B. (2003). Complexity in Teaching and Children's Mathematical Thinking. *International Group for the Psychology of Mathematics Education, 4*, 435-441.

Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics education*, *37*(3), 222-255

thank you